# UNIT-4

## Optimisation and Linear Programming

Ex. A company produces both interior and exterior paints from two raw materials $M_1$ & $M_2$

The following table provides the basic data of the problem

| | Ton of raw material per ton of Exterior Paints | Interior Paints | Max. daily availability (tons) |
|---|---|---|---|
| Raw Material ($M_1$) | 6 | 4 | 24 |
| Raw Material ($M_2$) | 1 | 2 | 6 |
| Profit/ton | 5 | 4 | |

A market survey indicates that the daily demand for interior paint cannot exceed that for the exterior paint by more than 1 ton. Also the max. demand for the interior paints is 2 tons.

the company wants to determine the optimum products (mix of both interior & exterior paints to maximise the total daily profit)

The model has three basic component —

① Precision variable that seek to determine i.e. we need to determine the daily amount to be produced a exterior & interior paints. Thus the variables of the model are defined as—

$x_1$ = tons produced daily of exterior paints

$x_2$ = " " " " interior "

② Objective (goal) is to optimise i.e. we need to maximise or minimise

③ constant that the solution must satisfy (sub. to the con

Objective that we need to optimize i.e the company wants to ~~optimize~~ maximize the daily profit given that the profit /ton of exterior paints & interior paints are 5 and respectively. Thus the total profit from exterior paints equal to $= 5x_1$ & Total profit from interior paint $= 4x_2$

$Z$ (Total daily profit) $= 5x_1 + 4x_2$

Hence the objective of the company is to maximise

$Z_{max} = 5x_1 + 4x_2$

Now the 3rd comp of the model is the const that restrict the material usage & product demand. The raw material restriction can be

$\left(\text{Usage of raw material by}\right) \le \left(\text{Max. raw material available}\right)$

The daily usage of raw material $M_1$ is 6 ton per ton of the exterior paint & 4 ton per ton of interior paint
Thus usage of raw material $M_1$ by ext. paint $= 6x_1$ tons/day
& usage of raw material $M_1$ by int. paint $= 4x_2$ tons/day
Hence, the total usage of raw material $M_1$ by both paints
$= (6x_1 + 4x_2)$ tons/day

Similarly,
The total usage of raw material $M_2$ by both paints $= (x_1 + 2x_2)$ ton/day

Now apply the availability limit we have -

$6x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 6$

The first demand restriction stimulates that the excess of daily production of interior over exterior paint

i.e. $x_2 - x_1$ should not exceed by 1 ton, which transl

$x_2 - x_1 \le 1$

$x_2 \le 2$

That is the market unit

The second demand restn stimulate that max. daily demand of int paint is limited to 2 tons
And implicit restn is that the variables $x_1$ & $x_2$ cann assume the negative value

$x_1 \ge 0$ ; $x_2 \ge 0$

LPP

$Z_{max} = 5x_1 + 4x_2$

Sub. to the condition

$6x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 6$

$x_2 - x_1 \le 1$

$x_2 \le 2$

$x_i \ge 0$ , $i = 1, 2, \ldots$

Ques 4 In a factory there are 3 types of machines say A, B and C and 4 types of product are manufactured say 1, 2, 3 & 4. We manufacturing a product it has to go through each type of machines. The time taken by each machine and total time in hour available for each ~~time~~ type of machine is given in the table below.

| Machine Type | Products 1 | 2 | 3 | 4 | Total time available in hours |
|---|---|---|---|---|---|
| A | 1.5 | 1 | 2.4 | 1 | 2000 |
| B | 1 | 5 | 1 | 3.5 | 8000 |
| C | 1.5 | 3 | 3.5 | 1 | 5000 |

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Profit/unit | 5.24 | 7.30 | 8.34 | 4.18 |

**Sol.** Suppose that the profit is proportional to th no. of product for each type of product

Let the no. of product for each type say 1, 2, 3, be $x_1, x_2, x_3$ & $x_4$ - The LPP model

$$Z_{max}(\text{Profit}) = 5.24x_1 + 7.30x_2 + 8.34x_3 + 4.18x_4$$

Subject to the constants

$$1.5x_1 + x_2 + 2.4x_3 + x_4 \leq 2000$$

$$x_1 + 5x_2 + x_3 + 8.5x_4 \leq 8000$$

$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 \leq 5000$$

$x_i \geq 0 \qquad i = 1, 2, 3, 4$

## Definitions

A function Z which is to be optimized (max or mi is called the objective $f^n$

If the objective $f^n$ (Z) and the subj. to the constan are the linear $f^n$ of $x_i$ such a programming is called linear programming.

Given a set of m linear inequalities or equation in r variables in which to find non-negativ value of these value which will satisfy the const & maximise or minimise some linear $f^n$ of

Mathematically we need to optimize

$$Z = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_rx_r \qquad \text{—— (1)}$$

Subject to the constants

$$a_{i_1}x_1 + a_{i_2}x_2 + \dots + a_{i_r}x_r \; (\leq, =, \geq) = b_i \qquad \text{—— (2)}$$

such that $x_i \geq 0$ $(i = 1, 2, \dots, r)$.

If the objective $f^n$ is not a linear $f^n$ of the variables or the $eq^n$ or inequalities are not linear in variables it is known as non-linear programm

Any set of variable say $x_j$ satisfying the $eq^n$ (2) is call the $sol^n$ of LPP.

Any $sol^n$ which satisfy the non-negative restriction is called feasible $sol^n$.

Any feasible $sol^n$ which optimize the objective $f^n$ is called optimal feasible $sol^n$.

## Graphical Method —

LPP which involves only 2 variables can be solved prominently by graphical method. The method includes 2 steps—

I → Determination of feasible $sol^n$ space

II → " " optimal $sol^n$ from among all th feasible point in space

$\frac{x_1}{5} + \frac{x_2}{3} \leq 1$

$\frac{x_1}{2} + \frac{x_2}{5} \leq 1$

Solve LPP graphically, Max $Z = 5x_1 + 3x_2$

subject to the constants ; $3x_1 + 5x_2 \leq 15$

and $5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$ $\quad 3\left(\frac{10-2x_2}{5}\right) + 5x_2$

$$3\left(\frac{10-2x_2}{5}\right) + 5x_2 \leq 15$$

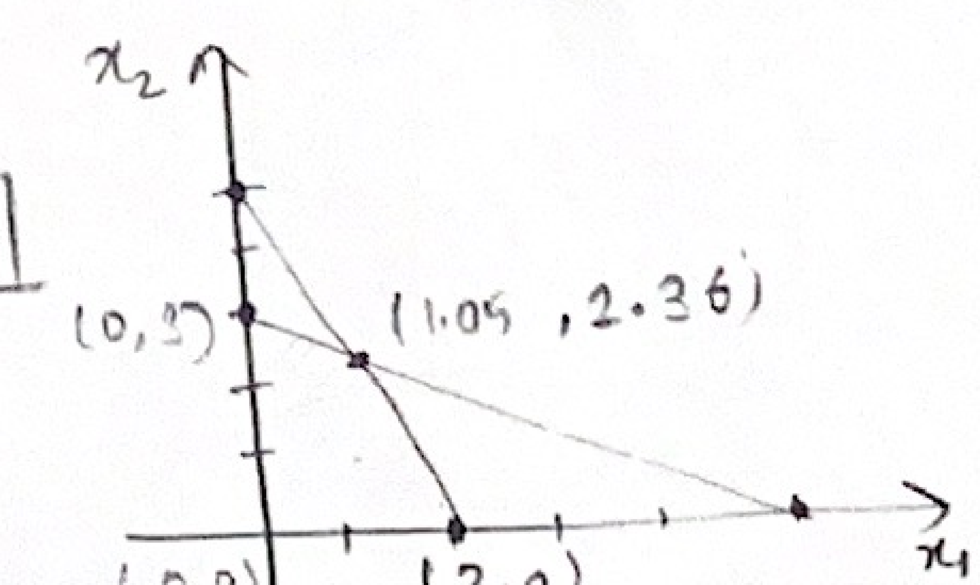
$$6 - \frac{6}{5}x_2 + 5x_2 \leq 15$$

$$\left(\frac{25-6}{5}\right)x_2 \leq 9$$

$$x_2 \leq \frac{45}{19}$$

$$\boxed{x_2 = 2.368}$$

$$\boxed{x_1 = 1.0528}$$



(0, 3) $Z = 9$

(2, 0) $Z = 10$

(1.05, 2.36) $Z = 12.33$

Q. Solve the LPP by graphical method, ~~maximise~~

Max $Z = 4x_1 + 5x_2$

Sub to constraints

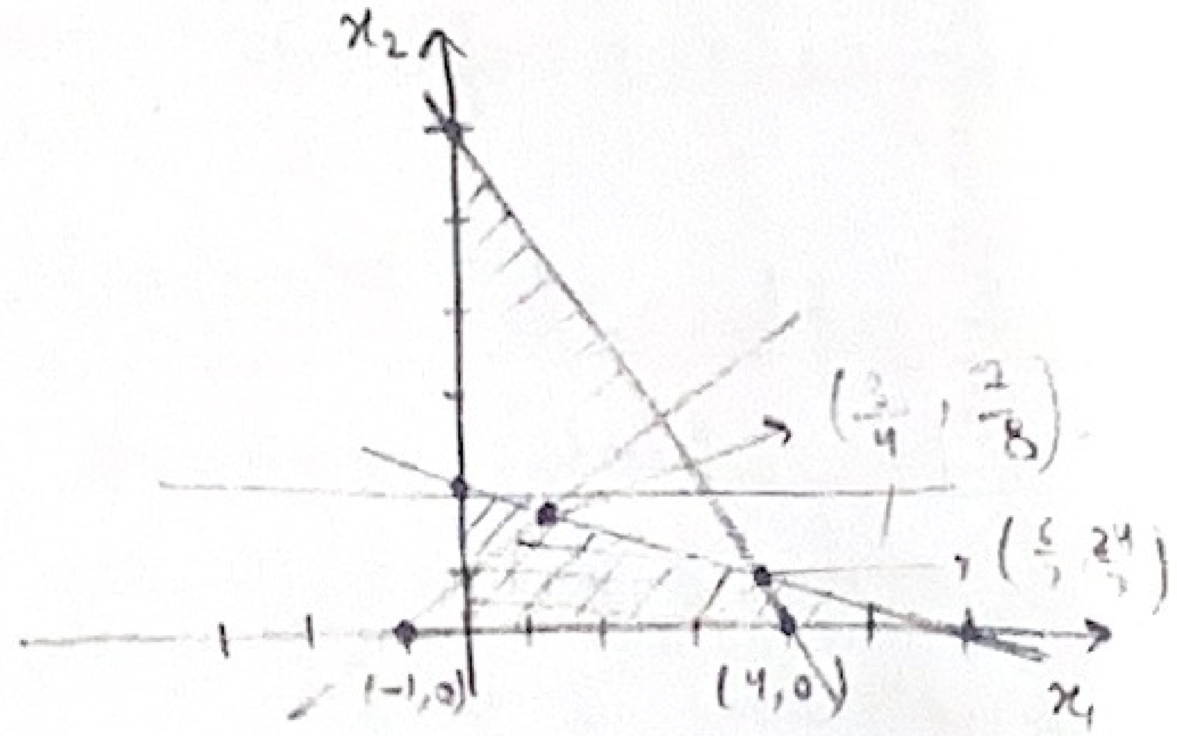
$$6x_1 + 4x_2 \le 24 \Rightarrow \frac{x_1}{4} + \frac{x_2}{6} \le 1$$

$$x_1 + 3x_2 \le 6 \Rightarrow \frac{x_1}{6} + \frac{x_2}{2} \le 1$$

$$-x_1 + x_2 \le 1$$

$$\& \quad x_2 \le 2$$

$$x_i \ge 0$$




$\frac{x_1}{6} + \left(\frac{1+x_1}{2}\right) = 1$

$x_1 + 3 + 3x_1 = 6$

$4x_1 = 3$

$\boxed{x_1 = \frac{3}{4}}$

$\boxed{x_2 = \frac{7}{8}}$

$Z = 4x_1 + 5x_2$

at $(\frac{3}{4}, \frac{7}{8})$

$Z = 4\left(\frac{3}{4}\right) + 5\left(\frac{7}{8}\right)$

$\boxed{Z = \frac{59}{8}}$ (7.375)

$6x_1 + 4x_2 = 24$

$6x_1 + 18x_2 = 36$

$-14x_2 = -12$

$\boxed{x_2 = \frac{6}{7}}$

$x_1 = 6 - 3\left(\frac{6}{7}\right)$

$\boxed{x_1 = \frac{24}{7}}$

at $(\frac{6}{7}, \frac{24}{7}) =$

$Z = \frac{24}{7} + \frac{120}{7}$

$\boxed{Z = \frac{144}{7}}$ (20.571) ✓

Q. Two products A and B to be manufactured. 1 single unit of product A requires 2.4 min of punch-press time & 5 min of assembly time. The profit ~~for~~ of the prod. is Rs 0.6 unit. One single unit of prod. B requires 3 ~~units~~ of punch-press time & 2.5 min welding time. The profit for the product B is Rs 0.70 per unit. The capacity of the punch department available for these products is 1200 min per week ~~unit~~. The welding dept. has capacity 600 min per week & the assembly dept has 1500 min per week. (i) Formulate the LPP method for model. (ii) Determine the quantity of prod A & B so that the profit is maximum

| Machine | Product A | Product B | Available Time (min/week) |
|---|---|---|---|
| Punch Press | 2.4 | 3 | 1200 |
| Welding | — | 2.5 | 600 |
| Assembly | 5 | — | 1500 |
| Profit (Rs/unit) | 0.60 | 0.70 | |

Let ~~max~~ & $x_1$ and $x_2$ be the no of products A & B respectively produced by the company.

Max $Z = 0.60x_1 + 0.70x_2$

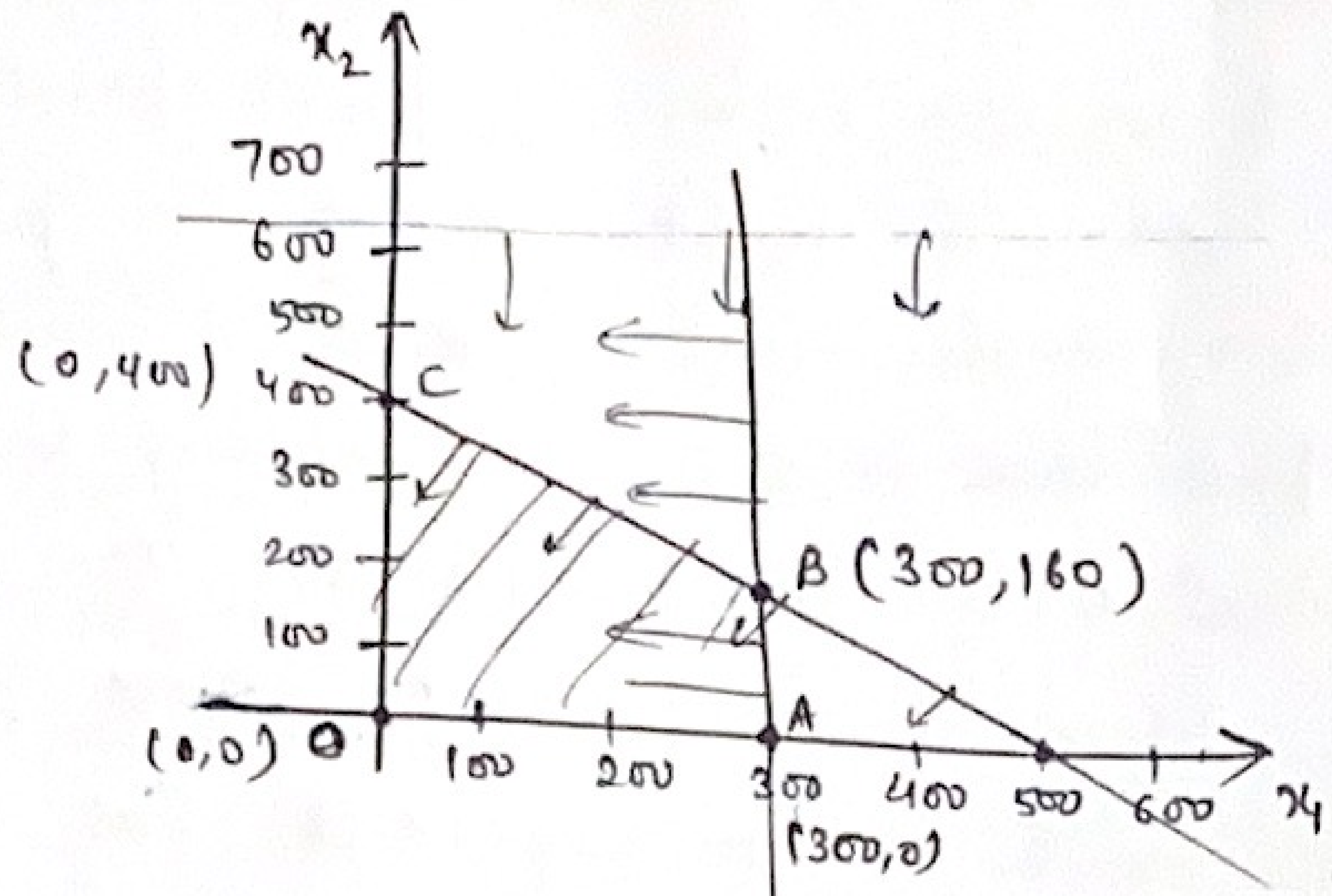
Sub to the constraints –

$$2.4x_1 + 3x_2 \le 1200$$

$$0 + 2.5x_2 \le 1600$$

$$5x_1 + 0 \le 1500$$

$$x_1, x_2 \ge 0$$




$2.4x_1 + 3x_2 = 1200$

$2.4(300) + 3x_2 = 1200$

$\boxed{x_2 = 160}$

$Z = 0.60x_1 + 0.70x_2$

at O (0,0)

$\boxed{Z = 0}$

at A (300,0)

$\boxed{Z = 180}$

at B (300,160)

$\boxed{Z = 292}$

at C (0,400)

$\boxed{Z = 280}$

Q Solve the LPP, $Z_{min} = 2x_1 + 3x_2$, subj. to the constrain

$$x_1 + x_2 \leq 4$$
$$6x_1 + 2x_2 \geq 8$$
$$x_1 + 5x_2 \geq 4$$
$$x_i \geq 0 \; ; \; i = 1,2$$

Q Solve the LPP, Max $Z = 2x_1 + 2x_2$

Sub. to the constraints →

$$x_1 - x_2 \geq -1$$
$$-0.5x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

{Unbounded ... There is no feasible region

Q A manufacturer of medicines is preparing a production plant. On machines A & B there are sufficient ingredient available to ~~sem~~ make 20,000 bottl of A and 40,000 bottles of B. But there are only 45,000 bottles availables into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough materials to fill 1000 bottles of medicine A & it takes 1 hr to prepare enough mat to fill 1000 of B & there are 66 hours available for thi. The profit ~~ooo~~ is Rs 1 /bottle for A & Rs 7/bott for B.

(i) Formulate the LPP problem.

(ii) How to schedule the prodn so as to get the max profit.

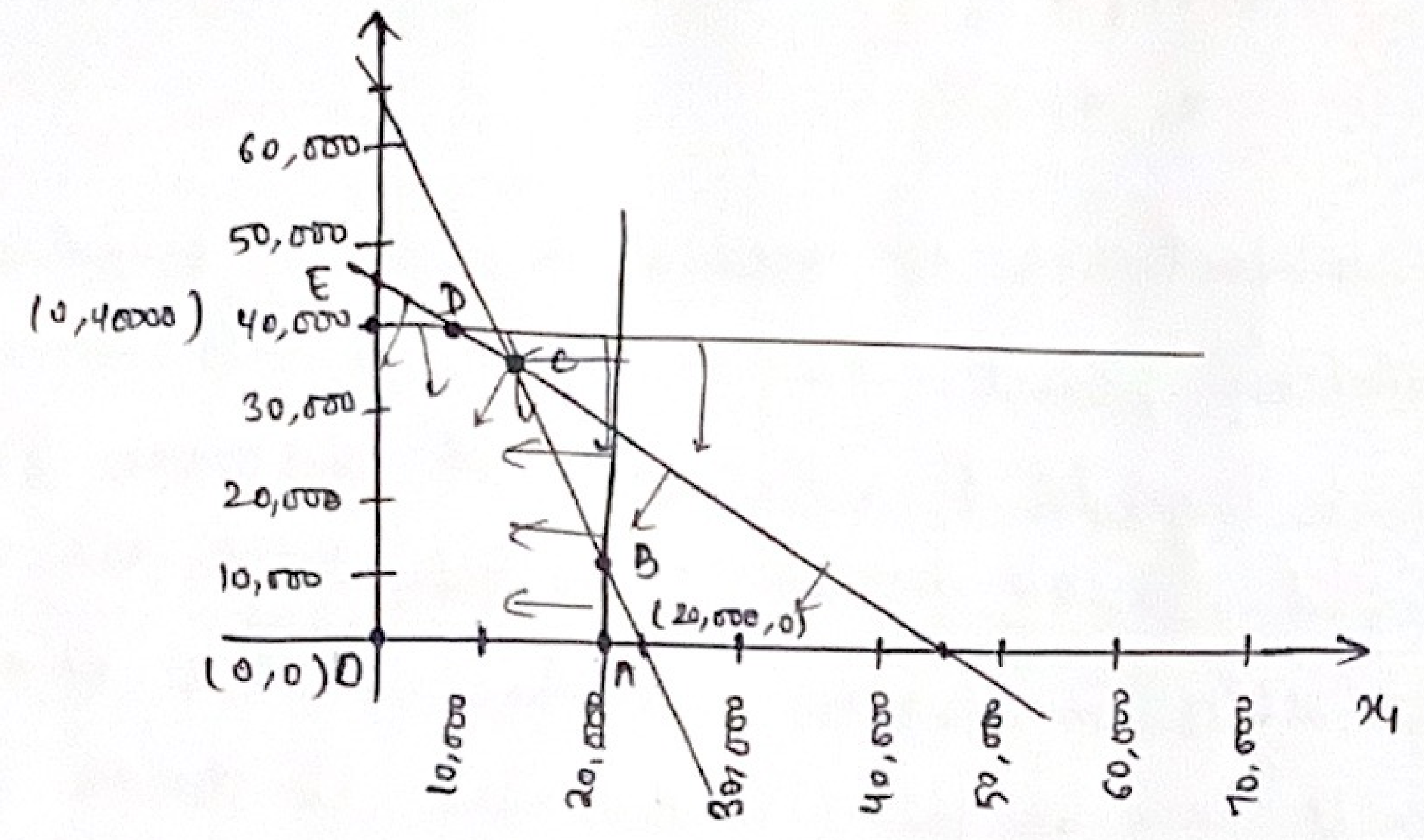
Let the no. of bottles be $x_1$ & $x_2$ be prepared for medicine A & B

$$Z_{max} = x_1 + 7x_2$$

Subj. to the constraints.

~~$20{,}000x_1 + 40000x_2 \leq 45000$~~

$$x_1 + x_2 \leq 45000$$
$$x_1 \leq 20000$$
$$x_2 \leq 40000$$
$$\frac{3}{1000}x_1 + \frac{1}{1000}x_2 \leq 66$$
$$3x_1 + x_2 \leq 66000$$
$$x_i \geq 0 \quad i = 1,2$$




$x_1 + x_2 = 45000$
$3x_1 + x_2 = 66000$
$\overline{-\quad -\quad\quad -}$
$-2x_1 = -21000$

$\boxed{x_1 = 10500}$

$\boxed{x_2 = 34500}$

C (10500, 34500)

$3x_1 + x_2 = 66000$
$x_1 = 20000$
$\boxed{x_2 = 6000}$

B (20000, 6000)

$x_1 + x_2 = 45000$
$x_2 = 40000$
$x_1 = 5000$

D (5000, 40000)

$Z_{max} = x_1 + 7x_2$

at O (0,0)
$\boxed{Z = 0}$

at A (20,000, 0)
$\boxed{Z = 20000}$

at B (20000, 6000)
$\boxed{Z = 62000}$

at C (10500, 34500)
$\boxed{Z = 252000}$

at C (5000, 40000)
$\boxed{Z = 285000}$

The max profit is 285000 attained at $x_1 = 5000$, $x_2 = 40000$

## Simplex Method

The simplex method is an iterative method which provides the solⁿ of any LPP in a finite no. of steps or give an indication that there is an unbounded solⁿ. In this method we move step by step from one extreme point to another extreme point till we reach the optimal extreme point. Consider the general LPP

$$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \; (\leq, =, \geq) = b_i$$
$$i = 1, 2, 3, \dots n$$

where one and only one sign less than, equal to, greater than equal on the RHS holds for each constant but the sign can vary from one const. to another:-

$$x_j \geq 0 \;;\; j = 1, 2, 3, \dots n$$
$$\text{Max } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

## Standard form or Normal form of LPP

Characl

1. All the const of a LPP are expressed in the form of an eqⁿ except the non-negative const which remai
2. The RHS of these constant eqⁿ is non-negative
3. All the decision variables are non-negative
4. The objective fⁿ is of maximization and minimization

The various steps involved in the computation of the optimal value of a LPP with the help of Simplex Method are as follows

**Step 1** → So that the objective T of the given LPP is maxi... If it is to be minimised then it should be conver into a prob. of maximization by using the

$$\text{Min}(z) = \text{Max}(-z)$$

i.e. Max $z'$ where $z' = -z$ the objective func. and its max. object

$$\text{Max } z' = T$$

$$\text{Min } z = -T$$

**Step 2** → Make all the $b_i$'s in the constant positive by multiplying the inequality by $-1$, if $b_i$'s in the const. is negative the inequality changes from $\geq$ to $\leq$ or $\leq$ to $\geq$

**Step 3** → Convert all the restricted equality into eq$^n$ by introducing non-negative / the constants and taking the corresponding $c_i$'s in the objective f$^n$ so that it is equal to zero

**Step 4** → Find the initial basic sol$^n$ to the problem in the form of

$$X_B = B^{-1} b$$

where $X_B$ is an $(m \times 1)$ column vector and b is some matrix of . Put this initial basic sol$^n$ $X_B$ in the first column of the simplex table

Compute the value of net evaluation ~~such that~~

$$Z_i - C_i \quad , \quad i = 1, 2, \ldots m$$

$$Z_i - C_i = C_B Y_i - C_i$$

where, $C_B = (C_{B_1}, C_{B_2}, \ldots C_{B_m})$

$C_{B_j}$ = jth comp of $C_B$ i.e. the comp. of C correspo to the variable $X_{B_j}$

and $Y_i = (y_{1i}, y_{2i}, \ldots y_{ji} \ldots y_{mi})$

where ~~the~~ each column vector $a_i$ in A is not equal to b

$$a_i = \sum_{j=1}^{m} y_{ji} b_j$$

Now examine the sign of $C_i - Z_i$

(i) If all $C_i - Z_i$ are less than equal to 0 then the initial basic i/p sol$^n$ $X_B$

(ii) If atleast one $C_i - Z_i > 0$ we proceed on the next step

**Step 6** → If more than one $C_i - Z_i$ is positive i.e. $Z_i - C_i < 0$ then choose among these which has the largest magnitude i.e. the most negative. Let it be

$$C_k - Z_k \text{ for some } i = k$$

① If all $y_{ik} \leq 0$ for j then there is an unbounded sol$^n$ to the given problem

② If atleast one $y_{jk} \geq 0$ then the corresponding vector $y_k$ enter the where, $Y_B = (Y_{B_1}, Y_{B_2}, \ldots Y_{B_m})'$ and we go to the next step.

**Step 7** → Choose the min of the ratio

$$\frac{X_{B_j}}{Y_{jk}}, \quad Y_{jk} > 0 \; ; \; j = 1, 2, \ldots m.$$

Let the min. be $\frac{X_{Br}}{Y_{rk}}$ then the vector $y_r$

The common element $y_{rk}$ which is in the r$^{th}$ row and k$^{th}$ column which is called leading element or pivotal element of the table.

**Step 8** → Change the leading element to unity by dividing its row in the table by that element itself & the remaining element in the column to zero by making use of

$$\hat{Y}_{ji} = Y_{ji} - \frac{Y_{ri}}{Y_{rK}} \cdot Y_{j \cdot K} \quad ; j = 1, 2, \ldots m+1, \; j \neq r$$

$$\hat{Y}_{ri} = \frac{Y_{ri}}{Y_{rK}} \quad , i = 0, 1, 2, \ldots m$$

**Step 9 →** Revert back to the step 5th and repeat the computation process until either an optimal solⁿ is reached and there's an indication of unbounded solⁿ

$\boxed{\text{Note}}$ If we anyhow know the matrix $B^{-1}$, where B is the basic matrix then the quantities can be computed -

$$X_B = B^{-1} b$$

$$Y_i = B^{-1} a_i \qquad i = 1, 2, \ldots, m+n$$

$$C_i - Z_i = C_i - C_B Y_i = C_i - C_B (B^{-1} a_i) \quad ; i = 1, 2, 3 \ldots$$

$$Z = C_B X_B = C_B (B^{-1} b)$$

Q. Solve the LPP by simplex Method

Max $Z = 7x_1 + 5x_2$

Subject to the constraints -

$x_1 + 2x_2 \leq 6$

$4x_1 + 3x_2 \leq 12$

$x_1, x_2 \geq 0$

The Normal form or Standard form of the given LPP

Max $Z = 7x_1 + 5x_2 + 0s_1 + 0s_2$

$x_1 + 2x_2 + s_1 + 0s_2 = 6$

$4x_1 + 3x_2 + 0s_1 + s_2 = 12$

[Don't use same slack variable in both eqⁿ]

Simplex Table-1

Pivotal column

enter

| Basic | Z | $x_1$ | $x_2$ | $s_1$ | $s_2$ | solⁿ | ratio |
|---|---|---|---|---|---|---|---|
| (z-row) Z | 1 | -7 | -5 | 0 | 0 | 0 | |
| ($s_1$-row) $s_1$ | 0 | 1 | 2 | 1 | 0 | 6 | $\frac{6}{1} = 6$ |
| ($s_2$-row) $s_2$ | 0 | 4 | 3 | 0 | 1 | 12 | $\frac{12}{4} = 3$ (m |

Pivotal row

leave

Pivotal element

For the simplex table -2 using the following transformation

New Z-row = current Z-row $- \left(-\frac{7}{4}\right) s_2$

New Z-row $= (1 \quad -7 \quad -5 \quad 0 \quad 0 \quad 0) + \frac{7}{4}(0 \quad 4 \quad 3 \quad 0 \quad 1 \quad 12)$

$= (1 \quad 0 \quad 1/4 \quad 0 \quad 7/4 \quad 21)$

New $s_1$-row = current $s_1$-row $- \left(\frac{1}{4}\right) s_2$

By using this transformation simplex-table-2 →

Simplex Table-2

| Basic | Z | $x_1$ | $x_2$ | $s_1$ | $s_2$ | solⁿ | ratio |
|---|---|---|---|---|---|---|---|
| Z | 1 | 0 | 1/4 | 0 | 7/4 | 21 | |
| $s_1$ | 0 | 0 | 5/4 | 1 | -1/4 | 3 | |
| $x_1$ | 0 | 4 | 3 | 0 | 1 | 12 | |

$x_1 = \frac{12}{4}$

$x_2 = 0$

$s_1 = \frac{3}{1} = 3$

$s_2 = 0$

Subs. these value in Max Z (objective ...)

$$\text{Max } Z = 7(3) + 5(0) = 21$$

Q Solve the following LPP by simplex Method-

$$\text{Max } Z = 2x_1 + 3x_2$$

<u>Subj. to constraints</u>-

$$2x_1 + x_2 \le 4$$
$$x_1 + 2x_2 \le 5$$
$$x_1, x_2 \ge 0$$

In the ratios consider only the +ve ratios

The normal form or standard form of the given LPP

$$\text{Max } Z = 2x_1 + 3x_2 + 0s_1 + 0s_2$$
$$2x_1 + x_2 + s_1 + 0s_2 = 4$$
$$x_1 + 2x_2 + 0s_1 + s_2 = 5$$

Transfer all the variable on LHS & consider the coeff of all the ...

<u>Simplex Table-1</u>

| | Basic | Z | $x_1$ | $x_2$ ← enter | $s_1$ | $s_2$ | sol$^n$ | ratio |
|---|---|---|---|---|---|---|---|---|
| (Z-row) | Z | 1 | -2 | -3 | 0 | 0 | 0 | (0) |
| ($s_1$-row) | $s_1$ | 0 | 2 | 1 | 1 | 0 | 4 | $4/1 = 4$ |
| ($s_2$-row) | $s_2$ | 0 | 1 | 2 | 0 | 1 | 5 | $5/2 = 2.5$ (min) |

Pivotal column  Pivotal element  Pivotal row  leave

For the $2^{nd}$ simplex table, by using the transformation

$$\text{New Z row} = \text{current Z row} - \left(\frac{-3}{2}\right) s_2$$
$$\text{New } s_1 \text{ row} = \text{current } s_1\text{-row} - \left(\frac{1}{2}\right) s_2$$

$\therefore$ New Z-row $= (1 \quad -2 \quad -3 \quad 0 \quad 0 \quad 0) + \frac{3}{2}(0 \quad 1 \quad 2 \quad 0 \quad 1 \quad 5)$

$= (1 \quad -1/2 \quad 0 \quad 0 \quad 3/2 \quad 15/2)$

New $s_1$-row $= (0 \quad 2 \quad 1 \quad 1 \quad 0 \quad 4) - \frac{1}{2}(0 \quad 1 \quad 2 \quad 0 \quad 1 \quad 5)$

$= (0 \quad \frac{3}{2} \quad 0 \quad 1 \quad \frac{-1}{2} \quad \frac{3}{2})$

<u>Simplex Table-2</u>

| | Basic | Z | $x_1$ ← enter | $x_2$ | $s_1$ | $s_2$ | sol$^n$ | rat |
|---|---|---|---|---|---|---|---|---|
| (Z-row) | Z | 1 | -1/2 | 0 | 0 | 3/2 | 15/2 | -15 |
| ($s_1$-row) | $s_1$ | 0 | 3/2 | 0 | 1 | -1/2 | 3/2 | 1 |
| ($x_2$-row) | $x_2$ | 0 | 1 | 2 | 0 | 1 | 5 | 5 |

Pivotal column  Pivotal element  Pivotal row

$$\text{New Z-row} = \text{current Z row} - \left(\frac{-1/2}{3/2}\right) s_1$$

$= (1 \quad -1/2 \quad 0 \quad 0 \quad 3/2 \quad 15/2) + \frac{1}{3}(0 \quad 3/2 \quad 0 \quad 1 \quad -1/2 \quad 3/2)$

$= (1 \quad 0 \quad 0 \quad 1/3 \quad 4/3 \quad 8)$

$$\text{New } x_2 \text{ row} = \text{current } x_2 - \left(\frac{1}{3/2}\right) s_1$$

$= (0 \quad 1 \quad 2 \quad 0 \quad 1 \quad 5) - \frac{2}{3}(0 \quad 3/2 \quad 0 \quad 1 \quad -1/2 \quad 3/2)$

$= (0 \quad 0 \quad 2 \quad -2/3 \quad 4/3 \quad 4)$

## Simplex Table - 3

| Basic | Z | $x_1$ | $x_2$ | $s_1$ | $s_2$ | sol$^n$ |
|---|---|---|---|---|---|---|
| Z | 1 | 0 | 0 | 1/3 | 4/3 | 8 |
| $x_1$ | 0 | 1 | 0 | 2/3 | -1/3 | 1 |
| $x_2$ | 0 | 0 | 2 | -2/3 | 4/3 | 4 |

$x_1 = \frac{1}{1} = 1$

$x_2 = \frac{4}{2} = 2$

$s_1 = s_2 = 0$

Max $Z = 2\times1 + 3\times2$

$= 2+6 = 8.$

Q Solve the following LPP by simplex method

Max $Z = 5x_1 + 4x_2$

Sub. to the constraints, -

$6x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 6$

$-x_1 + x_2 \le 1$

$x_2 \le 2$

$x_1, x_2 \ge 0$

The normal form or standard form of the given LPP

Max $Z = 5x_1 + 4x_2$

$6x_1 + 4x_2 + s_1 + 0s_2 + 0s_3 + 0s_4 = 24$

$x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 + 0s_4 = 6$

$-x_1 + x_2 + 0s_1 + 0s_2 + s_3 + 0s_4 = 1$

$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 2$

## Simplex Table - 1

Pivotal row

x1 ← enter

Pivotal element

| Basic | Z | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | sol$^n$ | ratio |
|---|---|---|---|---|---|---|---|---|---|
| Z | 1 | -5 | -4 | 0 | 0 | 0 | 0 | 0 | |
| $s_1$ | 0 | 6 | 4 | 1 | 0 | 0 | 0 | 24 | $\frac{24}{6} = 4$ min |
| $s_2$ | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 6 | $\frac{6}{1} = 6$ |
| $s_3$ | 0 | -1 | 1 | 0 | 0 | 1 | 0 | 1 | $\frac{1}{-1} = -1$ |
| $s_4$ | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 2 | |

Pivotal row

Pivotal element should be 1 (use transfn or leave it) but all other element other than pivotal element should be 0 in $x_1$ column (column which will enter)

Construct 2nd simplex table by using the transformation

New $Z$ = current $Z - \left(-\frac{5}{6}\right) s_1$

$= (1 \quad -5 \quad -4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0) + \frac{5}{6}$

$(0 \quad 6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 24)$

$-4 + \frac{20}{6}$

$-\frac{4}{6}$

$= (1 \quad 0 \quad -4/6 \quad 5/6 \quad 0 \quad 0 \quad 0 \quad 20)$

New $s_2$ = current $s_2 - \left(\frac{1}{6}\right) s_1$

$= (0 \quad 1 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 6) - \frac{1}{6}$

$(0 \quad 6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 24)$

$2 - \frac{4}{6} = \frac{8}{6}$

$= (0 \quad 0 \quad 8/6 \quad -1/6 \quad 1 \quad 0 \quad 0 \quad 2)$

New $s_3$ = current $s_3 - \left(-\frac{1}{6}\right) s_1$

$= (0 \quad -1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1) + \frac{1}{6}$
$(0 \quad 6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0, \quad 24)$

$= (0 \quad 0 \quad 10/6 \quad 1/6 \quad 0 \quad 1 \quad 0 \quad 5)$

New $s_4$ = current $s_4 - 0(s_1)$

$= (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1$

Simplex Table - 2

| Basic | Z | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | Sol$^n$ | ratio |
|---|---|---|---|---|---|---|---|---|---|
| Z | 1 | 0 | $-4/6$ | $5/6$ | 0 | 0 | 0 | 20 | $-30$ |
| $x_1$ | 0 | 6 | 4 | 1 | 0 | 0 | 0 | 24 | 6 |
| $s_2$ | 0 | 0 | $8/6$ | $-1/6$ | 1 | 0 | 0 | 2 | 1.5 min |
| $s_3$ | 0 | 0 | $10/6$ | $1/6$ | 0 | 1 | 0 | 5 | 3 |
| $s_4$ | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 2 | 2 min |

Pivotal col. enter

P.E

leave

leave

Construct 3rd simplex table using transformation

$20 + \frac{8}{6} = \frac{128}{6}$

$\therefore$ New Z = current Z $- \left(-\frac{1}{2}\right) s_2$

$(1 \quad 0 \quad -4/6 \quad 5/6 \quad 0 \quad 0 \quad 0 \quad 20$

$+\frac{1}{2} \, (0 \quad 0 \quad 4/3 \quad 0 \; -1/6 \quad 0 \quad 0 \quad 1 \quad 2$

$= 1 \quad 0 \quad 0 \quad 3/4 \quad \frac{1}{2} \quad 0 \quad 0 \quad 21$

New $x_1$ = current $x_1 - 3 s_2$

$(0 \quad 6 \quad 4 \quad 1 \quad 0 \quad 0 \quad 0 \quad 24) - 3$
$(0 \quad 0 \quad 8/6 \quad -1/6 \quad 1 \quad 0 \quad 0 \quad 2)$

$= (0 \quad 6 \quad 0 \quad 3/2 \quad -3 \quad 0 \quad 0 \quad 18)$

New $s_3$ = current $s_3 - \frac{5}{4} s_2$

$= (0 \quad 0 \quad 10/6 \quad 1/6 \quad 0 \quad 1 \quad 0 \quad 5)$

$-\frac{5}{4} (0 \quad 0 \quad 8/6 \quad -1/6 \quad 1 \quad 0 \quad 0 \quad 2)$

$= (0 \quad 0 \quad 0 \quad 3/8 \quad -5/4 \quad 1 \quad 0 \quad \frac{5}{2})$

New $s_4$ = current $s_4 - \frac{6}{8} s_2$

$= (0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 2)$

$-\frac{6}{8} (0 \quad 0 \quad 8/6 \quad -1/6 \quad 1 \quad 0 \quad 0 \quad 2)$

$= (0 \quad 0 \quad 0 \quad 1/8 \quad -6/8 \quad 0 \quad 1 \quad 1/2)$

## Simplex Table – 5

| Basic | $Z$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | Solⁿ |
|---|---|---|---|---|---|---|---|---|
| $Z$ | 1 | 0 | 0 | 3/4 | 1/2 | 0 | 0 | 21 |
| $x_1$ | 0 | 6 | 0 | 3/2 | −3 | 0 | 0 | 18 |
| $x_2$ | 0 | 0 | 8/6 | −1/6 | 1 | 0 | 0 | 2 |
| $s_3$ | 0 | 0 | 0 | 3/8 | −5/4 | 1 | 0 | 5/2 |
| $s_4$ | 0 | 0 | 0 | 1/8 | −6/8 | 0 | 1 | 1/2 |

$x_1 = \frac{18}{6} = 3$; $x_2 = \frac{2\times 6}{8} = \frac{3}{2}$; $s_3 = \frac{5}{2}$; $s_4 = 1/2$

$s_1 = s_2 = 0$

$\text{Max } z = 5(3) + 4\left(\frac{3}{2}\right) + 0(0) + 0(0) + 0(1) + 0(1)$

$= 15 + 6$

$= 21$

**Q.** Solve LPP by Simplex Method

$\text{Max } Z = x_1 + 2x_2 + x_3$

Subj to constraints –

$2x_1 + x_2 - x_3 \le 2$

$-2x_1 + x_2 - 5x_3 \ge -6$ —— (1)

$4x_1 + x_2 + x_3 \le 6$

$x_1, x_2, x_3 \ge 0$

~~Normal or Standard form of this LPP~~

Multiply by $(-1)$ in eqⁿ (1) $\Rightarrow 2x_1 - x_2 + 5x_3 \le 6$

Now, Normal or standard form of this LPP

$\text{Max } Z = x_1 + 2x_2 + x_3$

$2x_1 + x_2 - x_3 + s_1 + 0s_2 + 0s_3 = 2$

$+2x_1 - x_2 + 5x_3 + 0s_1 + s_2 + 0s_3 = 6$

$4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

## Simplex Table – 1

Pivotal column (enter): $x_2$

| Basic | $Z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | solⁿ | ratio |
|---|---|---|---|---|---|---|---|---|---|
| $Z$ | 1 | −2 | −2 | −1 | 0 | 0 | 0 | 0 | |
| $s_1$ | 0 | 2 | 1 | −1 | 1 | 0 | 0 | 2 | 2 |
| $s_2$ | 0 | 2 | −1 | 5 | 0 | 1 | 0 | 6 | −6 |
| $s_3$ | 0 | 4 | 1 | 1 | 0 | 0 | 1 | 6 | 6 |

Pivotal row: $s_1$ (P.E. = 1)

Const 2nd simplex table –

New $Z$ = current $Z - \left(\frac{-2}{1}\right) s_1$

$= (1 \quad -1 \quad -2 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0)$

$+2(0 \quad 2 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2)$

$= (1 \quad 3 \quad 0 \quad -3 \quad 2 \quad 0 \quad 0 \quad 4)$

New $s_2$ = current $s_2 - (-1) s_1$

$= (0 \quad 2 \quad -1 \quad 5 \quad 0 \quad 1 \quad 0 \quad 6)$

$+1(0 \quad 2 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2)$

$= (0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$

New $s_3$ = current $s_3 - (1) s_1$

$= (0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 6)$

$-1(0 \quad 2 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2)$

$= (0 \quad 2 \quad 0 \quad 2 \quad -1 \quad 0 \quad 1 \quad 4)$

## Simplex Table – 2

Pivotal column (enter): $x_3$

| Basic | $Z$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | solⁿ | ratio |
|---|---|---|---|---|---|---|---|---|---|
| $Z$ | 1 | 3 | 0 | −3 | 2 | 0 | 0 | 4 | −4/ |
| $x_2$ | 0 | 2 | 1 | −1 | 1 | 0 | 0 | 2 | −2 |
| $s_2$ | 0 | 4 | 0 | 4 | 1 | 1 | 0 | 8 | $\frac{8}{4} = 2$ |
| $s_3$ | 0 | 2 | 0 | 2 | −1 | 0 | 1 | 4 | $\frac{4}{2} = 2$ |

P.E. = 4 (row $s_2$, leave)

Pivotal row

## Contd 3rd Simplex table

ratio → min positive; 2+3 until all entries z row are -ve & (0, -ve)

New $Z$ = current $Z - \left(\frac{-3}{4}\right) s_2$

$$= (1 \quad 3 \quad 0 \quad -3 \quad 2 \quad 0 \quad 0 \quad 4)$$
$$+\frac{3}{4}(0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$$
$$= (1 \quad 6 \quad 0 \quad 0 \quad 11/4 \quad 3/4 \quad 0 \quad 10)$$

New $x_2$ = current $x_2 - \left(\frac{-1}{4}\right) s_2$

$$= (0 \quad 2 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2)$$
$$+\frac{1}{4}(0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$$
$$= (0 \quad 3 \quad 1 \quad 0 \quad 5/4 \quad 1/4 \quad 0 \quad 4)$$

New $s_3$ = current $s_3 - \left(\frac{2}{4}\right) s_2$

$$= (0 \quad 2 \quad 0 \quad 2 \quad -1 \quad 0 \quad 1 \quad 4)$$
$$-\frac{1}{2}(0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$$
$$= (0 \quad 0 \quad 0 \quad 0 \quad -3/2 \quad -1/2 \quad 1/2 \quad 0)$$

### Simplex Table 3

| Basic | Z | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | sol$^n$ | ratio |
|---|---|---|---|---|---|---|---|---|---|
| Z | 1 | 6 | 0 | 0 | 11/4 | 3/4 | 0 | 10 | |
| $x_2$ | 0 | 3 | 1 | 0 | 5/4 | 1/4 | 0 | 4 | |
| $x_3$ | 0 | 4 | 0 | 4 | 1 | 1 | 0 | 8 | |
| $s_3$ | 0 | 0 | 0 | 0 | -3/2 | -1/2 | 1/2 | 0 | |

$x_2 = \frac{4}{1} = 4$ ; $x_3 = \frac{8}{4} = 2$ ; $x_1 = 0$ ; $s_3 = 0$

Max Z = $x_1 + 2x_2 + x_3$

$= 0 + 2(4) + 2$ ... $= 8$

---

**Sol Q.** Solve the following LPP by the simplex Method –

Min $Z = -x_1 - 2x_2 - x_3$

Subject to constraints –

$2x_1 + x_2 - x_3 \leq 2$

$-2x_1 + x_2 - 5x_3 \geq -6$

$4x_1 + x_2 + x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

The normal form or the standard form of the given LPP is –

Min $Z = -x_1 - 2x_2 - x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to the constraints

$2x_1 + x_2 - x_3 + s_1 + 0s_2 + 0s_3 = 2$

$+2x_1 - x_2 + 5x_3 + 0s_1 + s_2 + 0s_3 = +6$

$4x_1 + x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 6$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

This is the 1st non-zero entry in the pivotal row, it is to be reduced to unity

### Simplex Table-I

Pivotal column (enter)

| Basic | Z | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | sol$^n$ | ratio |
|---|---|---|---|---|---|---|---|---|---|
| Z | 1 | 1 | 2 | 1 | 0 | 0 | 0 | -0 | |
| $s_1$ | 0 | 2 | 1 | -1 | 1 | 0 | 0 | 2 | $\frac{2}{1} = 2$ |
| $s_2$ | 0 | 2 | -1 | 5 | 0 | 1 | 0 | 6 | $\frac{6}{-1}$ (neglect) |
| $s_3$ | 0 | 4 | 1 | 1 | 0 | 0 | 1 | 6 | $\frac{6}{1} = 6$ |

PE

Pivotal row

For the next Simplex Table II using the transf$^n$ –

New Z = current Z $- \left(\frac{2}{1}\right) s_1$

$$= (1 \quad 1 \quad 2 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)$$
$$2(0 \quad 2 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2)$$
$$= (1 \quad -3 \quad 0 \quad 3 \quad -2 \quad 0 \quad 0 \quad -4)$$

New $s_2$ = current $s_2 - \left(\frac{-1}{1}\right) s_1$

= current $s_2 + s_1$

$z\,(0 \quad 2 \quad -1 \quad 5 \quad 0 \quad 1 \quad 0 \quad 6)\;+$

$1\,(0 \quad 2 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2)$

$\text{New } s_2 = (0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$

New $s_3$ row = current $s_3 - (1)\,s_1$

$= (0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 6)$

$-1\,(0 \quad 2 \quad 1 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2)$

$= (0 \quad 2 \quad 0 \quad 2 \quad -1 \quad 0 \quad 1 \quad 4)$

Simplex Table - II

| Basic | Z | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | sol$^n$ | ratio |
|---|---|---|---|---|---|---|---|---|---|
| Z | 1 | $-3$ | 0 | 3 | $-2$ | 0 | 0 | $-4$ | |
| $x_2$ | 0 | 1 | $1/2$ | $-1/2$ | $1/2$ | 0 | 0 | 1 | $-2$ |
| $s_2$ | 0 | 4 | 0 | 4 | 1 | 1 | 0 | 8 | 2 |
| $s_3$ | 0 | 2 | 0 | 2 | $-1$ | 0 | 1 | 4 | 2 |

Remark.

Reduce First non-zero entry in the pivotal row to unity

For the next simple Table III using the transf$^n$ -



New Z-row = current $Z - \left(\frac{3}{4}\right) s_2$

$= (1 \quad -3 \quad 0 \quad 3 \quad -2 \quad 0 \quad 0 \quad -4)\;-$

$\frac{3}{4}\,(0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$

$= (1 \quad -6 \quad 0 \quad 0 \quad -11/4 \quad -3/4 \quad 0 \quad -10)$

~~= (1 −3/2 0 0 −11/4 −3/4 0 −10)~~

New $x_2$-row = current $x_2 - \left(\frac{-\frac{1}{2}}{4}\right) s_2$

$= (0 \quad 1 \quad 1/2 \quad -1/2 \quad 1/2 \quad 0 \quad 0 \quad 1)$

$+\frac{1}{8}\,(0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$

$= (0 \quad 3/2 \quad 1/2 \quad 0 \quad 5/8 \quad 1/8 \quad 0 \quad 2)$

New $s_3$ row = current $s_3 - \left(\frac{2}{4}\right) s_2$

$= (0 \quad 2 \quad 0 \quad 2 \quad -1 \quad 0 \quad 1 \quad 4)\;-$

$\frac{1}{2}\,(0 \quad 4 \quad 0 \quad 4 \quad 1 \quad 1 \quad 0 \quad 8)$

$= (0 \quad 0 \quad 0 \quad 0 \quad -3/2 \quad -1/2 \quad 1 \quad 0)$

∴ ~~Simplex Table III~~

~~Basic~~ Reduce the first non-zero entry (ie 4) in pivotal column ($s_2$) to unity (4 is the first non-zero entry in the pivotal row)

New $s_2$ = current $s_2/4$

New $s_2 = (0 \quad 1 \quad 0 \quad 1 \quad 1/4 \quad 1/4 \quad 0 \quad 2)$

Simplex Table III

| Basic | Z | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | sol$^n$ | rat |
|---|---|---|---|---|---|---|---|---|---|
| Z | 1 | $-6$ | 0 | 0 | $-11/4$ | $-3/4$ | 0 | $-10$ | |
| $x_2$ | 0 | $3/2$ | $1/2$ | 0 | $5/8$ | $1/8$ | 0 | 2 | |
| $x_3$ | 0 | 1 | 0 | 1 | $1/4$ | $1/4$ | 0 | 2 | |
| $s_3$ | 0 | 0 | 0 | 0 | $-3/2$ | $-1/2$ | 1 | 0 | |

$x_2 = \frac{2}{1/2} = 4$ ; $x_3 = \frac{2}{1} = 2$ ; $s_3 = 0$

$x_1 = 0, \; s_1 = 0, \; s_2 = 0$

$\text{Min } z = -x_1 - 2x_2 - x_3 + 0s_1 + 0s_2 + 0s_3$

$= 0 - 2(4) - 2 = -10.$

Q.Y Solve the following LPP by simplex Method

$$\text{Max } Z = -2x_1 - x_2 + 5x_3$$

Subj to the constraints

$$x_1 - 2x_2 + x_3 \leq 8$$

$$3x_1 - x_2 \geq -18$$

$$2x_1 + x_2 - 2x_3 \leq -4$$

$$x_1, x_2, x_3 \geq 0$$